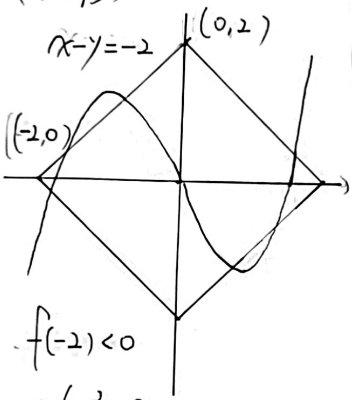
114竹北

7. 若 $a>0$，函數 $y=f(x)=x^3-3a^2x$ 的圖形與方程式 $|x|+|y|=2$ 之圖形，有 6 個相異之交點，則 a 之範圍為__________。

$\left(\sqrt{\frac{2}{3}}, \frac{2}{\sqrt{3}}\right)$



$f(-2)<0$

$\Rightarrow 6a^2<8$

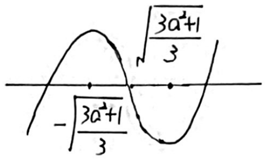
$\begin{cases} y=x+2 \\ y=x^3-3a^2x \end{cases}$ 2交點

令 $g(x)=x^3-(3a^2+1)x-2$

$g'(x)=3x^2-(3a^2+1)=0$

$\Rightarrow x^2=\dfrac{3a^2+1}{3}$

$g(x)=x(x^2-(3a^2+1))-2x^2$



$g\left(-\sqrt{\frac{3a^2+1}{3}}\right)$

$=-(-2)\left(\sqrt{\frac{3a^2+1}{3}}\right)^3-2>0$

$\Rightarrow 3a^2+1>3$

$\Rightarrow \frac{2}{3}<a^2<\frac{4}{3} \Rightarrow \sqrt{\frac{2}{3}}<a<\frac{2}{\sqrt{3}}$

$g\left(\sqrt{\frac{3a^2+1}{3}}\right)$

$=-2\left(\sqrt{\frac{3a^2+1}{3}}\right)^3-2<0$

8. 已知數列 $\{a_n\}$ 滿足 $a_1=2, a_{n+1}=\dfrac{2^{n+1}a_n}{\left(n+\frac{1}{2}\right)a_n+2^n}$，設 $b_n=\dfrac{2^n}{a_n}$，則 $b_n=$________。（以 n 表示）

$\frac{n^2+1}{2}$

$2^{n+1}\cdot a_n=2^n\cdot a_{n+1}+(n+\frac{1}{2})a_n a_{n+1}$

$\Rightarrow b_{n+1}=b_n+n+\frac{1}{2}$

$b_2=b_1+1+\frac{1}{2}$

$b_3=b_2+2+\frac{1}{2}$

$\vdots$

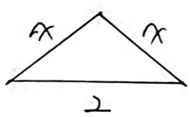
$+)\ b_n=b_{n-1}+(n-1)+\frac{1}{2}$

$\Rightarrow b_n=b_1+\dfrac{n\cdot(n-1)}{2}+\dfrac{n-1}{2}$

$=\dfrac{n^2+1}{2}$

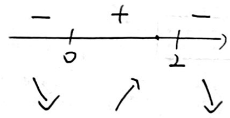
9. $\triangle ABC$ 中，$\overline{AB}=\overline{AC}$，$\overline{BC}=2$，$\angle BAC=2\theta$，$0<\theta<\frac{\pi}{2}$，若 $\triangle ABC$ 之外接圓半徑為 R，內切圓半徑為 r，則 $\dfrac{r}{R}$ 之最大值為______。

$\frac{1}{2}$



$\Delta=\sqrt{x^2-1}$

$\dfrac{r}{R}=\dfrac{\frac{\Delta}{x+1}}{\frac{2x^2}{4\Delta}}=2\cdot\dfrac{x^2-1}{x^2(x+1)}=2\left(\dfrac{x-1}{x^2}\right)$



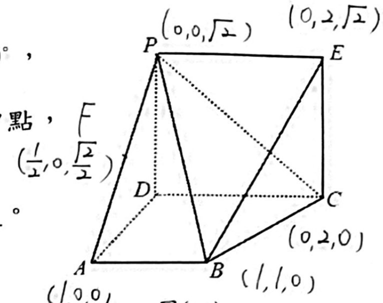
$\frac{d}{dx}(\ \ )=0 \Rightarrow x^2=(x-1)\cdot 2x$

$\Rightarrow x=0$ or 2

$\Rightarrow max=2\cdot\frac{2-1}{4}=\frac{1}{2}$

10. 如右圖(二)，$\overline{PD}$ 垂直於梯形 $ABCD$ 所在的平面，$\angle ADC = \angle BAD = 90^\circ$，$\overline{PD} = \sqrt{2}$，$\overline{AB} = \overline{AD} = \frac{1}{2}\overline{CD} = 1$，四邊形 $PDCE$ 為矩形，若 F 為 $\overline{PA}$ 中點，Q 為 $\overline{EF}$ 上一點，且 $\overline{BQ}$ 與平面 BCP 所夾的角為 $\frac{\pi}{6}$，則 $\overline{FQ} =$ ________。

$\frac{\sqrt{19}}{2}$



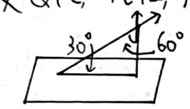
圖(二)

$\begin{matrix} 1 & -\sqrt{2} & 1 & 1 & -\sqrt{2} \\ 2 & -\sqrt{2} & 0 & 2 & -\sqrt{2} \end{matrix}$

$\vec{n} /\!/ (\sqrt{2}, \sqrt{2}, 2) /\!/ (1,1,\sqrt{2})$

$\overrightarrow{EF} /\!/ (\frac{1}{2}, -2, -\frac{\sqrt{2}}{2}) /\!/ (1,-4,-\sqrt{2})$

設 $Q(t, -4t+2, -\sqrt{2}t+\sqrt{2})$



$\overrightarrow{BQ} \cdot \vec{n} = (t-1, -4t+1, -\sqrt{2}t+\sqrt{2}) \cdot (1,1,\sqrt{2})$

$= -5t+2 = 2 \cdot \frac{1}{2} \cdot \sqrt{19t^2-14t+4}$

$\Rightarrow 6t^2 - 6t = 0 \Rightarrow t = 1$

$\Rightarrow Q(1,-2,0)$

$\Rightarrow \overline{FQ} = \frac{\sqrt{19}}{2}$

11. 已知函數 $f(x)$ 的定義域是實數 R，且 $f(x+2)-2$ 是奇函數，$f(2x+1)$ 是偶函數。若 $f(1) = 0$，試求 $f(1) + f(2) + \cdots + f(2025) =$ ________。

4048

$f(-x+2) + f(x+2) = 4$

$f(2) + f(2) = 4$

$f(1) + f(3) = 4$

$f(0) + f(4) = 4$

$f(-1) + f(5) = 4$

$f(-2) + f(6) = 4$

$f(-2x+1) = f(2x+1)$

$f(0) = f(2)$

$f(-1) = f(3)$

$f(-2) = f(4)$

$f(-3) = f(5)$

$f(-4) = f(6)$

0, 2, 4, 2, 0, 2, 4, 2, …

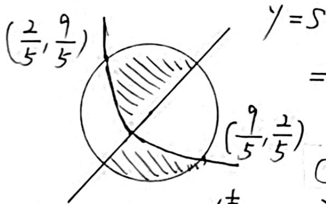
$8 \cdot 506 + 0 = 4048$

12. 設 (x,y) 滿足 $\begin{cases} (y-x)\left(y-\dfrac{18}{25x}\right) \geq 0 \\ (x-1)^2+(y-1)^2 \leq 1 \end{cases}$，則 $2x-y$ 的最小值為________。

-1

令 $x = C+1$，$y = S+1$ 代入 $xy = \frac{18}{25}$

$\Rightarrow S + C + SC = \frac{-7}{25}$



	C	S
猜	$\frac{-3}{5}$	$\frac{4}{5}$
or	$\frac{4}{5}$	$\frac{-3}{5}$

$2 \cdot \frac{2}{5} - \frac{9}{5} = -1$

$\frac{1}{5} + \left(\frac{-12}{25}\right) = \frac{-7}{25}$