

$$pf: \text{令 } \vec{OA}=(m,n,0), \vec{OB}=(p,q,0), \vec{OC}=(1,1,1)$$

$$\text{考慮 } \vec{OA}, \vec{OB} \text{ 夾角 } 45^\circ \Rightarrow \sqrt{m^2+n^2} \sqrt{p^2+q^2} \times \frac{\sqrt{2}}{2} = mp+nq \dots (1)$$

$$\text{考慮 } \vec{OB}, \vec{OC} \text{ 夾角 } 45^\circ \Rightarrow \sqrt{p^2+q^2} \sqrt{3} \times \frac{\sqrt{2}}{2} = p+q \dots (2)$$

$$\text{考慮 } \vec{OA}, \vec{OC} \text{ 夾角 } 45^\circ \Rightarrow \sqrt{m^2+n^2} \sqrt{3} \times \frac{\sqrt{2}}{2} = m+n \dots (3)$$

$$\text{又 } \sqrt{m^2+n^2}=1, \sqrt{p^2+q^2}=1$$

$$|\vec{AB}|^2 = |\vec{OA}|^2 + |\vec{OB}|^2 - 2|\vec{OA}||\vec{OB}|\cos\theta$$

$$\Rightarrow \cos\theta = \frac{mp+nq}{\sqrt{m^2+n^2}\sqrt{p^2+q^2}} = mp+nq = \frac{\sqrt{2}}{2}, \theta = \pi/4$$