$$3.w=\cos\frac{2\pi}{n}+i\sin\frac{2\pi}{n},A_n=(\frac{5}{4}-\frac{w^2+1}{2w})(\frac{5}{4}-\frac{w^4+1}{2w^2})(\frac{5}{4}-\frac{w^6+1}{2w^6})......(\frac{5}{4}-\frac{w^{2n-2}+1}{2w^{n-1}})$$

令$Z^n=1,w,w^2,...,w^{n-1}\Rightarrow Z^n-1=(z-1)(z-w)(z-w^2)...(z-w^{n-1})=(z-1)(1+z+z^2+...+z^{n-1})$

$$\Rightarrow(1+z+z^2+...+z^{n-1})=(z-w)(z-w^2)...(z-w^{n-1})$$

考慮$A_n=(\frac{5}{4}-\frac{w^2+1}{2w})(\frac{5}{4}-\frac{w^4+1}{2w^2})(\frac{5}{4}-\frac{w^6+1}{2w^6})......(\frac{5}{4}-\frac{w^{2n-2}+1}{2w^{n-1}})$

$$=(\frac{5}{4}-\frac{w}{2}-\frac{1}{2w})(\frac{5}{4}-\frac{w^2}{2}-\frac{1}{2w^2})(\frac{5}{4}-\frac{w^3}{2}-\frac{1}{2w^3})......(\frac{5}{4}-\frac{w^{n-1}}{2}-\frac{1}{2w^{n-1}})$$

$$=\frac{1}{4^{n-1}}(5-2w-\frac{2}{w})(5-2w^2-\frac{2}{w^2})(5-2w^3-\frac{2}{w^3})(5-2w^{n-1}-\frac{2}{w^{n-1}})$$

$$=\frac{(-1)^{n-1}}{4^{n-1}}(\frac{2w^2-5w+2}{w})(\frac{2w^4-5w^2+2}{w^2})(\frac{2w^6-5w^3+2}{w^3})......(\frac{2w^{2n-2}-5w^{n-1}+2}{w^{n-1}})$$

$$=\frac{(-1)^{n-1}}{4^{n-1}}(\frac{(w-2)(2w-1)}{w})(\frac{(w^2-2)(2w^2-1)}{w^2})(\frac{(w^3-2)(2w^3-1)}{w^3})......(\frac{(w^{n-1}-2)(2w^{n-1}-1)}{w^{n-1}})$$

$$=\frac{(-1)^{n-1}}{4^{n-1}}(w-2)(w^2-2)(w^3-2)......(w^{n-1}-2)(2w-1)(2w^2-1)(2w^3-1)...(2w^{n-1}-1)\times\frac{1}{w^{\frac{n(n-1)}{2}}}$$

$$=\frac{(-1)^{n-1}}{4^{n-1}}(2^n-1)(2^n)\times(1-(\frac{1}{2})^n)\times\frac{1}{w^{\frac{n(n-1)}{2}}}$$

$$A_n=\frac{(-1)^{n-1}}{4^{n-1}}(2^n-1)(2^n)\times(1-(\frac{1}{2})^n)\Rightarrow\lim_{n\to\infty}A_n=(-1)^{n-1}\times4=???$$

其中z=2帶入$(1+z+z^2+...+z^{n-1})=(z-w)(z-w^2)...(z-w^{n-1})$

$$\Rightarrow(w-2)(w^2-2)(w^3-2)......(w^{n-1}-2)=(-1)^{n-1}(2^n-1)$$

$$(2w-1)(2w^2-1)(2w^3-1)...(2w^{n-1}-1)=2^{n-1}(w-\frac{1}{2})(w^2-\frac{1}{2})(w^3-\frac{1}{2})...(w^{n-1}-\frac{1}{2})$$

$$=(-1)^{n-1}2^{n-1}(\frac{1}{2}-w)(\frac{1}{2}-w^2)(\frac{1}{2}-w^3)......(\frac{1}{2}-w^{n-1})$$

$$=(-1)^{n-1}2^{n-1}(1+\frac{1}{2}+(\frac{1}{2})^2+...+(\frac{1}{2})^{n-1})$$

$$=(-1)^{n-1}2^n(1-(\frac{1}{2})^n)$$

$$\frac{1}{w^{\frac{n(n-1)}{2}}}=1$$