計算3

條件:$a\ge\frac{-3}{2},b\ge\frac{-3}{2},c\ge\frac{-3}{2},d\ge\frac{-3}{2},e\ge\frac{-3}{2},f\ge\frac{-3}{2},a^5+b^5+c^5+d^5+e^5+f^5=2$

求$a^2+b^2+c^2+d^2+e^2+f^2<n,n$是最小正整數,求$n$?

pf:因為$a\ge\frac{-3}{2},b\ge\frac{-3}{2},c\ge\frac{-3}{2},d\ge\frac{-3}{2},e\ge\frac{-3}{2},f\ge\frac{-3}{2}\Rightarrow a+b+c+d+e+f+9\ge0$

令$f(a,b,c,d,e,f)=a^2+b^2+c^2+d^2+e^2+f^2$,

$g(a,b,c,d,e,f)=a^5+b^5+c^5+d^5+e^5+f^5-2$

$$\begin{cases}\nabla f(a,b,c,d,e,f)=2a\overrightarrow{x_1}+2b\overrightarrow{x_2}+2c\overrightarrow{x_3}+2d\overrightarrow{x_4}+2e\overrightarrow{x_5}+2f\overrightarrow{x_6}\\ \nabla g(a,b,c,d,e,f)=5a^4\overrightarrow{x_1}+5b^4\overrightarrow{x_2}+5c^4\overrightarrow{x_3}+5d^4\overrightarrow{x_4}+5e^4\overrightarrow{x_5}+5f^4\overrightarrow{x_6}\end{cases}$$

因為$\nabla f=\lambda\nabla g\Rightarrow\frac{2a}{5a^4}=\frac{2b}{5b^4}=\frac{2c}{5c^4}=\frac{2d}{5d^4}=\frac{2e}{5e^4}=\frac{2f}{5f^4}=\lambda$

$\Rightarrow a=b=c=d=e=f=(\frac{2}{5\lambda})^{\frac{1}{3}}$代入$a+b+c+d+e+f+9\ge0\Rightarrow\lambda\le\frac{-16}{135}$

再最後帶入$f(a,b,c,d,e,f)$前,檢查條件$a\ge\frac{-3}{2},b\ge\frac{-3}{2},c\ge\frac{-3}{2},d\ge\frac{-3}{2},e\ge\frac{-3}{2},f\ge\frac{-3}{2}$

是否有遺漏,在比較寬鬆的條件下,用Lagrange得到$a=(\frac{2}{5\lambda})^{\frac{1}{3}}$

接著檢查$a=(\frac{2}{5\lambda})^{\frac{1}{3}}\ge\frac{-3}{2}\Rightarrow(\frac{2}{5\lambda})^{\frac{1}{3}}\ge\frac{-3}{2}\Rightarrow\lambda\le\frac{-16}{135}$(結果一樣!!)

最後考慮$a^2+b^2+c^2+d^2+e^2+f^2=(\frac{2}{5\lambda})^{\frac{2}{3}}\times6\le\frac{27}{2}$,所以$n=14$