

$$4. a_{n+2} = a_{n+1} - a_n$$

特徵方程式： $X^2 - X + 1 = 0$

$$X = \frac{1 + \sqrt{3}i}{2} = \omega \text{ or } X = \frac{1 - \sqrt{3}i}{2} = \frac{1}{\omega}, \quad \omega + \frac{1}{\omega} = 1, \quad \omega^2 = \omega - 1,$$

$$\omega^3 = -1$$

$$\text{令 } a_n = C_1 \omega^n + C_2 \left(\frac{1}{\omega}\right)^n$$

$$\begin{cases} C_1 \omega + C_2 \frac{1}{\omega} = 1 \\ C_1 \omega^2 + C_2 \left(\frac{1}{\omega}\right)^2 = 1 \end{cases}$$

$$\text{解得 } C_1 = \frac{\omega^2 + 1}{\omega^2 - 1}, \quad C_2 = \frac{\omega - 1}{\omega + 1}$$

$$a_n = \frac{\omega^2 + 1}{\omega^2 - 1} \omega^n + \frac{\omega - 1}{\omega + 1} \left(\frac{1}{\omega}\right)^n$$