

新北市立樟樹國際實創高級中等學校 107 學年度教師甄選筆試

**【教育專業科目】及【學校行政實務與班級經營管理】試題(數學科)**

第三、四大題參考解答

三、參考解答

$$1.(1) A = \begin{bmatrix} \frac{1}{2} & \frac{7}{10} \\ \frac{1}{2} & \frac{3}{10} \end{bmatrix}, x_{n+1} = Ax_n \Rightarrow \begin{bmatrix} p_{n+1} \\ q_{n+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{7}{10} \\ \frac{1}{2} & \frac{3}{10} \end{bmatrix} \begin{bmatrix} p_n \\ q_n \end{bmatrix}, q_n = 1 - p_n$$

$$\Rightarrow p_{n+1} = \frac{1}{2}p_n + \frac{7}{10} - \frac{7}{10}p_n = -\frac{1}{5}p_n + \frac{7}{10} \Rightarrow \begin{cases} p_1 = \frac{1}{3} \\ p_{n+1} = -\frac{1}{5}p_n + \frac{7}{10} (n \geq 1) \end{cases}$$

$$(2) p_{n+1} - \alpha = -\frac{1}{5}(p_n - \alpha) \Rightarrow \alpha = \frac{7}{12} \Rightarrow p_{n+1} - \frac{7}{12} = -\frac{1}{5}(p_n - \frac{7}{12})$$

$$\Rightarrow p_n - \frac{7}{12} = (p_1 - \frac{7}{12})\left(-\frac{1}{5}\right)^{n-1} \Rightarrow p_n = \left(-\frac{1}{5}\right)^{n-1} + \frac{7}{12}$$

$$2.(284,20)=4, a_1 = a_{21} = a_{41} = \dots = a_{281} = a_{17} = a_{37} = \dots = a_{277} = a_{13} \\ = a_{33} = \dots = a_{273} = a_9 = a_{29} = \dots = a_{269} = a_5$$

所以此數列為 4 個一循環,  $a_{19} = a_3 = -2$ ,  $a_{71} = a_1 = 10$ ,  $a_{140} = a_4 = 7$

$$\text{令 } a_{230} = a_2 = x \Rightarrow 5(-2 + 10 + 7 + x) = 30 \Rightarrow a_{230} = -9$$

$$3. \frac{3 \times 2 \times 3}{3^7} = \frac{2}{243}$$

$$4. (a + \log_4 3)^2 = (a + \log_2 3)(a + \log_8 3)$$

$$\Rightarrow a^2 + 2a \log_4 3 + (\log_4 3)^2 = a^2 + (\log_2 3 + \log_8 3)a + \log_2 3 \times \log_8 3$$

$$a = -\frac{1}{4} \log_2 3 \Rightarrow r = \frac{a + \log_4 3}{a + \log_2 3} = \frac{1}{3}$$

四、參考解答

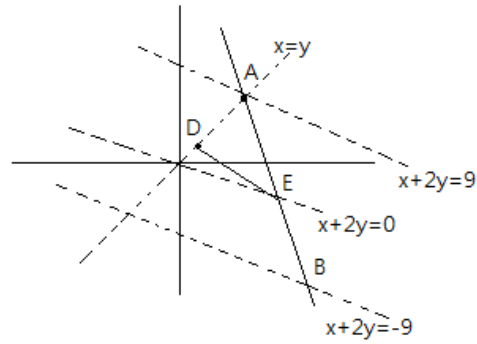
$$1. \text{設 } \overline{AB} = \overline{AC} = x \Rightarrow \overrightarrow{BD} \cdot \overrightarrow{AC} = (\overrightarrow{AD} - \overrightarrow{AB}) \cdot \overrightarrow{AC} = -\frac{1}{2}$$

$$\Rightarrow \frac{1}{2}x^2 - x \cdot x \cdot \frac{x^2 + x^2 - 4}{2} = -\frac{1}{2} \Rightarrow x^2 = 5 \quad \text{又} \quad \overrightarrow{CE} \cdot \overrightarrow{AB} = (\overrightarrow{AE} - \overrightarrow{AC}) \cdot \overrightarrow{AB} = -\frac{4}{3}$$

2.

$$\begin{cases} x = y \\ x + 2y = -9 \end{cases} \Rightarrow x = -3$$

$$\therefore -3 \leq \alpha < 1 \Rightarrow (m, n) = (-3, 1)$$



3. 設

$$\frac{x+y+2}{x-y+2} = k \Rightarrow (k-1)x - (k+1)y + 2k - 2 = 0$$

$$\Rightarrow \frac{|k-1-k-1+2k-2|}{\sqrt{(k-1)^2 + (k+1)^2}} \leq 1 \Rightarrow 1 \leq k \leq 7 \therefore 1 \leq \frac{x+y+2}{x-y+2} \leq 7$$

$$4. \int_{-4}^5 \pi(25 - x^2) dx = 162\pi$$