

$$a^2b^2 + b^2c^2 + c^2d^2 + d^2a^2 = (a^2 + c^2)(b^2 + d^2) \leq \left(\frac{a^2 + c^2 + b^2 + d^2}{2} \right)^2 = 4$$

$$\left(\frac{a^2}{b} + \frac{b^2}{c} + \frac{c^2}{d} + \frac{d^2}{a} \right) \left(\frac{a^2}{b} + \frac{b^2}{c} + \frac{c^2}{d} + \frac{d^2}{a} \right) (a^2b^2 + b^2c^2 + c^2d^2 + d^2a^2) \geq (a^2 + b^2 + c^2 + d^2)^3 = 64$$

$$\left(\frac{a^2}{b} + \frac{b^2}{c} + \frac{c^2}{d} + \frac{d^2}{a} \right) \left(\frac{a^2}{b} + \frac{b^2}{c} + \frac{c^2}{d} + \frac{d^2}{a} \right) \geq 16$$

$$\frac{a^2}{b} + \frac{b^2}{c} + \frac{c^2}{d} + \frac{d^2}{a} \geq 4$$