

$$1. \because X, Y \in \mathbb{R}, X^2 + Y^2 = 25$$

$$\therefore X = 5\cos\theta, \quad 0 \leq \theta \leq \frac{\pi}{2} \quad (\text{極坐標轉換})$$

$$Y = 5\sin\theta$$

$$5X^2 + 4XY + Y^2$$

$$= 5(5\cos\theta)^2 + 4(5\cos\theta)(5\sin\theta) + (5\sin\theta)^2 \quad \text{用二倍角和半角公式}$$

$$= 125\cos^2\theta + 100\cos\theta\sin\theta + 25\sin^2\theta \quad (\sin 2\theta = 2\sin\theta\cos\theta)$$

$$= 125 \times \frac{1+\cos 2\theta}{2} + 50\sin 2\theta + 25 \times \frac{1-\cos 2\theta}{2} \quad (\cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}})$$

$$= 75 + 50\cos 2\theta + 50\sin 2\theta \quad (\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}})$$

$$= 75 + 50\sqrt{2}\sin(2\theta + \frac{\pi}{4}) \quad (\text{用 SIN 合角公式疊合})$$

$$\text{又 } \frac{\pi}{4} \leq 2\theta + \frac{\pi}{4} \leq \frac{5\pi}{4} \quad \text{即 } -\frac{\sqrt{2}}{2} \leq \sin(2\theta + \frac{\pi}{4}) \leq \frac{\sqrt{2}}{2}$$

因此最小值為 $75 - 50 = 25$ ·ANS