

$$\begin{aligned}
& \cos^2(a-b) + \cos^2(b-c) + \cos^2(c-a) \\
&= \frac{\cos(2a-2b)+1}{2} + \frac{\cos(2b-2c)+1}{2} + \frac{\cos(2c-2a)+1}{2} \\
&= \frac{1}{2} \left[\cos 2a \cos 2b + \sin 2a \sin 2b + \cos 2b \cos 2c + \sin 2b \sin 2c + \cos 2c \cos 2a + \sin 2c \sin 2a + \frac{1}{2} (\sin^2 2a + \cos^2 2a + \sin^2 2b + \cos^2 2b + \sin^2 2c + \cos^2 2c) + \frac{3}{2} \right] \\
&= \frac{1}{2} \left[\frac{1}{2} (\sin 2a + \sin 2b + \sin 2c)^2 + \frac{1}{2} (\cos 2a + \cos 2b + \cos 2c)^2 + \frac{3}{2} \right]
\end{aligned}$$

當 $\sin 2a + \sin 2b + \sin 2c = 0, \cos 2a + \cos 2b + \cos 2c = 0$ 時，有最小值 $\frac{3}{4}$

可取 $a = \frac{\pi}{3}, b = -\frac{\pi}{3}, c = 0$