令 $f(x) = (x-2)(Ax+B)$

$\quad f(f(x)) = [(x-2)(Ax+B)-2][A(x-2)(Ax+B)+B]$

$\quad f(f(x)) = [Ax^2 + (-2A+B)x - 2B - 2][A^2x^2 + (-2A^2+AB)x - 2AB + B]$

(1) $Ax^2 + (-2A+B)x - 2B - 2 = 0$　有重根4

$$\Rightarrow \begin{cases} \alpha + \beta = -\dfrac{(-2A+B)}{A} = 8 \\ \alpha\beta = \dfrac{-2B-2}{A} = 16 \end{cases} \Rightarrow \begin{cases} -2A+B = -8A \\ B+1 = -8A \end{cases} \Rightarrow A = -\dfrac{1}{2}, B = 3$$

$$\Rightarrow f(x) = (x-2)(-\frac{1}{2}x+3) = -\frac{1}{2}(x-2)(x-6)$$

檢查 $f(f(x)) = -\dfrac{1}{2}[-\dfrac{1}{2}(x-2)(x-6)-2][-\dfrac{1}{2}(x-2)(x-6)-6]$

$\qquad f(f(x)) = -\dfrac{1}{2}[-\dfrac{1}{2}(x^2-8x+12)-2][-\dfrac{1}{2}(x^2-8x+12)-6]$

$\qquad f(f(x)) = -\dfrac{1}{2}[-\dfrac{1}{2}(x^2-8x+16)][-\dfrac{1}{2}(x^2-8x+24)]$　$x = 4,4$(重根)

$\therefore f(x) = (x-2)(-\dfrac{1}{2}x+3) = -\dfrac{1}{2}(x-2)(x-6)$

$\therefore f(0) = -6$　（無答案 ??? ）

(2) $[A^2x^2 + (-2A^2+AB)x - 2AB + B = 0$　有重根 4

$$\Rightarrow \begin{cases} \alpha + \beta = -\dfrac{(-2A^2+AB)}{A^2} = 8 \\ \alpha\beta = \dfrac{-2AB+B}{A^2} = 16 \end{cases} \Rightarrow \begin{cases} -2A^2+AB = -8A^2 \\ B(-2A+1) = 16A^2 \end{cases} \Rightarrow A = -\dfrac{3}{2}, B = 9$$

$$\Rightarrow f(x) = (x-2)(-\frac{3}{2}x+9) = -\frac{3}{2}(x-2)(x-6)$$

檢查 $f(f(x)) = -\dfrac{3}{2}[-\dfrac{3}{2}(x-2)(x-6)-2][-\dfrac{3}{2}(x-2)(x-6)-6]$

$\qquad f(f(x)) = -\dfrac{3}{2}[-\dfrac{3}{2}(x^2-8x+12)-2][-\dfrac{3}{2}(x^2-8x+12)-6]$

$\qquad f(f(x)) = -\dfrac{3}{2}[-\dfrac{1}{2}(3x^2-24x+40)][-\dfrac{3}{2}(x^2-8x+16)]$

$\qquad x = 4,4,\dfrac{12\pm2\sqrt{6}}{3}$(有四根 不合)