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$$\text{令 } x = 10^{100}$$

x^{200} 除以 $x+3$ 的餘數為 $(-3)^{200} = 3^{200}$

$$10^{20000} = (10^{100} + 3) \times \left[\frac{10^{20000}}{10^{100} + 3} \right] + 3^{200}$$

$$10^{20000} \equiv 0 \pmod{10}$$

$$10^{100} + 3 \equiv 3 \pmod{10}$$

$$3^{200} \equiv 1 \pmod{10}$$

$$\left[\frac{10^{20000}}{10^{100} + 3} \right] \equiv 3 \pmod{10}$$

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另解

$$\angle BDE = \angle CDF = x$$

$$\angle CFD = \angle AFE = y$$

$$\angle AEF = \angle BED = z$$

$$\pi = \angle EDF + \angle DFE + \angle FED = \pi - 2x + \pi - 2y + \pi - 2z = 3\pi - 2(x + y + z)$$

$$x + y + z = \pi$$

$$\angle A = x, \angle B = y, \angle C = z$$

$\triangle ABC, \triangle AFE, \triangle DBE, \triangle DFC$ 均相似

$$\overline{BD} = 6a, \overline{BE} = 5a, \overline{AF} = 6b, \overline{AE} = 4b, \overline{CF} = 5c, \overline{CD} = 4c$$

$$5a + 4b = 6$$

$$6b + 5c = 4$$

$$4c + 6a = 5$$

$$a = \frac{3}{4}, b = \frac{9}{16}, c = \frac{1}{8}$$

$$\text{所求} = 4a + 5b + 6c = \frac{105}{16}$$