

第 21 題

$$16^t - 9^t = 12^t$$

$$\text{let } a = 4^t, b = 3^t$$

$$a^2 - b^2 = ab$$

$$a^2 - ba - b^2 = 0$$

$$a = \left(\frac{1 + \sqrt{5}}{2} \right) b$$

$$\frac{a}{b} = \left(\frac{4}{3} \right)^t = \frac{1 + \sqrt{5}}{2}$$

$$\frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \left(\frac{3}{4} \right)^t + \left(\frac{4}{3} \right)^t = \frac{2}{1 + \sqrt{5}} + \frac{1 + \sqrt{5}}{2} = \sqrt{5}$$

第 24 題

設直線 BF 和直線 AD 交於 Q

作 QR 垂直直線 BE 於 R

$$\frac{\overline{BE}}{\overline{BR}} = \frac{\overline{FE}}{\overline{QR}}$$

$$\frac{14}{14 + \overline{ER}} = \frac{6}{8}$$

$$\overline{ER} = \frac{14}{3}$$

$$\frac{\overline{DP}}{\overline{EP}} = \frac{\overline{DQ}}{\overline{EB}} = \frac{\frac{32}{3}}{14} = \frac{16}{21}$$

$$\Delta BDP = \frac{16}{37} \Delta BDE = \frac{16}{37} \times 56 = 24 \frac{8}{37}$$

$$a + b + c = 69$$

第 25 題

$$\left(\frac{c}{a+b} \right)^2 = \frac{c^2}{c^2 + 2ab} \geq \frac{c^2}{c^2 + a^2 + b^2} = \frac{1}{2}$$

$$\frac{c}{a+b} \geq \frac{\sqrt{2}}{2}$$

第 26 題

應該只有 78 個乘積吧？

第 28 題

$$\overline{AB} = 10x + y, \overline{CD} = 10y + x$$

$$1 \leq x \leq 9, 0 \leq y \leq 9$$

$$\overline{OE} = t$$

$$\left(\frac{10x+y}{2} - t\right)\left(\frac{10x+y}{2} + t\right) = \left(\frac{10y+x}{2}\right)^2$$

$$t^2 = \left(\frac{10x+y}{2}\right)^2 - \left(\frac{10y+x}{2}\right)^2$$

$$t = \frac{3}{2}\sqrt{11(x+y)(x-y)} \in \mathcal{Q}$$

$$x + y = 11, x - y = 1$$

$$x = 6, y = 5$$

$$\overline{AB} = 65$$

第 29 題

$$\Delta BQC = \frac{1 - \frac{1}{421}}{4} = \frac{105}{421}$$

作 EM 垂直 BQ 於 M

$$\overline{BE} = x = \frac{1}{n}$$

$$\frac{x}{\overline{BC}} = \frac{\overline{ME}}{\overline{QC}}$$

$$\overline{QC} = \frac{1}{\sqrt{421}x}$$

$$\Delta BQC = \frac{1}{2} \times \frac{1}{\sqrt{421x}} \times \left(\frac{1}{\sqrt{421x}} - \frac{1}{\sqrt{421}} \right) = \frac{105}{421}$$

$$x = \frac{1}{15}$$

$$n = 15$$

第 30 題

$$\overline{AQ} = x, \overline{BQ} = 1, \overline{AP} = a, \overline{DP} = b, \overline{BC} = a + b$$

$$ax = b(x + 1) = a + b$$

$$x = \frac{a + b}{a} = \frac{a}{b}$$

$$a = \left(\frac{1 + \sqrt{5}}{2} \right) b$$

$$\frac{\overline{AQ}}{\overline{BQ}} = x = \frac{1 + \sqrt{5}}{2}$$