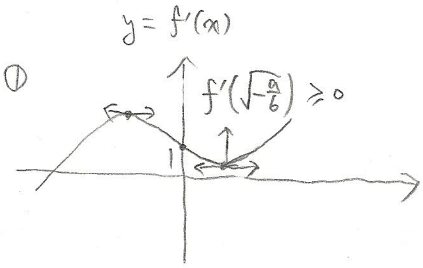
5. $f(x) = x^4 + ax^2 + x + 1$ 在 $[0,\infty)$ 遞增 —①



$\Rightarrow f'(x) = 4x^3 + 2ax + 1 \geq 0 \quad , \forall x \geq 0$

$\Rightarrow f''(x) = 12x^2 + 2a = 2(6x^2 + a) = 0$

$\Rightarrow x^2 = -\frac{a}{6} \Rightarrow x = \pm\sqrt{-\frac{a}{6}}$

(1) 若 $a \geq 0$，①式恆成立

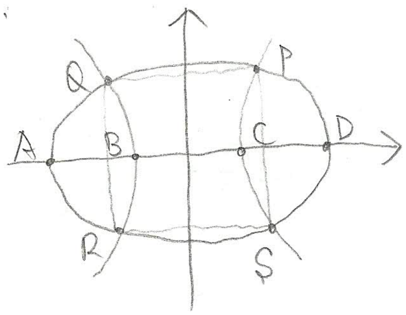
(2) 若 $a<0$，如圖，則 $f'(-\sqrt{\frac{a}{6}}) = \frac{4}{3}a(-\sqrt{\frac{a}{6}}) + 1 \geq 0$

$\Rightarrow 0 < -\frac{4}{3}a\sqrt{-\frac{a}{6}} \leq 1 \quad \Rightarrow \frac{\cancel{16}^{8}}{9}a^2\left(-\frac{a}{\cancel{6}_{3}}\right) \leq 1 \quad \Rightarrow a^3 \geq \frac{-27}{8} = \left(-\frac{3}{2}\right)^3$

$\Rightarrow -\frac{3}{2} \leq a < 0$

(1) $\cup$ (2) 得 $a \geq -\frac{3}{2}$ ※

6.



設橢圓 eq: $\frac{x^2}{a^2} + \frac{y^2}{a^2-c^2} = 1$, $P(1,1)$

$\Rightarrow$ 雙曲線 eq: $\frac{x^2}{c^2} - \frac{y^2}{a^2-c^2} = 1$

$$\Rightarrow \begin{cases} \frac{1}{a^2} + \frac{1}{a^2-c^2} = 1 \\ \frac{1}{c^2} - \frac{1}{a^2-c^2} = 1 \end{cases} \Rightarrow \begin{cases} \frac{1}{a^2} + \frac{1}{c^2} = 2 \text{，即 } a^2+c^2 = 2a^2c^2 \\ a^2-c^2+a^2 = a^4-a^2c^2 \quad \text{—①} \\ a^2-c^2-c^2 = a^2c^2-c^4 \quad \text{—②} \end{cases}$$

①+②: $3(a^2-c^2) = a^4-c^4 \Rightarrow a^2+c^2 = 3$, $a^2c^2 = \frac{3}{2}$

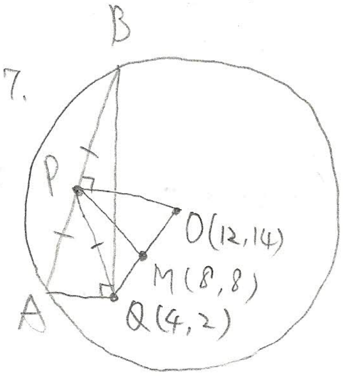
$\Rightarrow a^2(3-a^2) = \frac{3}{2} \Rightarrow 2a^4 - 6a^2 + 3 = 0$

$\Rightarrow a^2 = \frac{6 \pm \sqrt{12}}{4} = \frac{3}{2} \pm \frac{\sqrt{3}}{2}$ ($\because a>c \therefore$ 負不合)

$\Rightarrow a^2 = \frac{3+\sqrt{3}}{2}$, $c^2 = \frac{3-\sqrt{3}}{2}$

$\therefore \frac{\overline{BC}}{\overline{AD}} = \frac{2c}{2a} = \frac{c}{a} = \sqrt{\frac{3-\sqrt{3}}{3+\sqrt{3}}} = \sqrt{\frac{12-6\sqrt{3}}{6}} = \sqrt{2-\sqrt{3}} = \frac{\sqrt{6}-\sqrt{2}}{2}$ ※

7.



$C: (x-12)^2 + (y-14)^2 = 376$

設 $O(12,14)$, $M(8,8)$ 為 $\overline{OQ}$ 中點 , $P(x,y)$

由中線定理 及 $\overline{OP} \perp \overline{AB}$

$\overline{PM}^2 = \frac{1}{4}\left[2\left(\overline{PO}^2 + \overline{PQ}^2\right) - \overline{OQ}^2\right] = \frac{1}{4}\left(2\overline{OB}^2 - \overline{OQ}^2\right)$

$\Rightarrow (x-8)^2 + (y-8)^2 = \frac{1}{4}(2 \cdot 376 - 208)$

$\Rightarrow (x-8)^2 + (y-8)^2 = 136$ ※