101 北一女

1. 設 $Q(1+t,\ k+2t,\ 4+3t) \in L$ ，又 $P(1,0,3)$

$$\Rightarrow \overline{PQ} = \sqrt{t^2+(2t+k)^2+(3t+1)^2}$$
$$= \sqrt{14t^2+(6+4k)t+k^2+1}$$
$$= \sqrt{14\left[t^2-\frac{3+2k}{7}t+\left(\frac{3+2k}{14}\right)^2\right]+k^2+1-\frac{(3+2k)^2}{14}}$$
$$\leq \sqrt{\frac{5}{7}k^2-\frac{6}{7}k+\frac{5}{14}}$$
$$= \sqrt{\frac{5}{7}\left(k^2-\frac{6}{5}k+\frac{9}{25}\right)+\frac{5}{14}-\frac{9}{35}}$$
$$\leq \sqrt{\frac{7}{70}} = \frac{\sqrt{10}}{10}$$

$\therefore\ d(P,L)$ 的 $\min = \frac{\sqrt{10}}{10}$ ※

2. $f(x) = \dfrac{\frac{x^{101}-1}{x-1}+z}{x^{101}}$ ， $z = \cos\frac{2\pi}{17}+i\sin\frac{2\pi}{17}$

$$\Rightarrow f(1+\bar{z}) = \frac{\frac{(1+\bar{z})^{101}-1}{\bar{z}}+z}{(1+\bar{z})^{101}} = \frac{(1+\bar{z})^{101}-1+z\bar{z}}{\bar{z}(1+\bar{z})^{101}} = \frac{1}{\bar{z}} = z = \cos\frac{2\pi}{17}+i\sin\frac{2\pi}{17}$$ ※

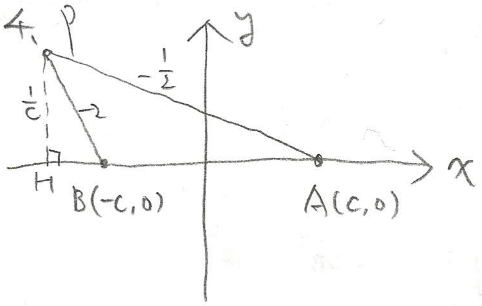
3. $S_n = 3+5+\cdots+(2n+1) = (n+1)^2-1 = n(n+2)$

$$\Rightarrow f(n) = \frac{n(n+2)}{(n+25)(n+2)(n+4)} = \frac{n}{n^2+29n+100} > 0$$

先考慮 $\frac{n^2+29n+100}{n} = 29+n+\frac{100}{n} \geq 29+2\sqrt{n\cdot\frac{100}{n}} = 49$

$\therefore\ \frac{n}{n^2+29n+100} \leq \frac{1}{49}$ $\therefore\ f(n)$ 之 $\text{Max} = \frac{1}{49}$ ※

4.



$\frac{1}{2}\cdot 2c\cdot\overline{PH} = 1 \Rightarrow \overline{PH} = \frac{1}{c}$

$\Rightarrow \overline{BH} = \frac{1}{2c}$ ， $\overline{AH} = \frac{2}{c}$

$\Rightarrow \frac{2}{c}-\frac{1}{2c} = 2c$ ， $\frac{3}{2c} = 2c$ ， $2c = \sqrt{3}$ ， $c = \frac{\sqrt{3}}{2}$

$\Rightarrow \overline{PB} = \sqrt{5}\,\overline{BH} = \sqrt{5}\cdot\frac{1}{\sqrt{3}} = \frac{\sqrt{5}}{\sqrt{3}}$

$\overline{PA} = \sqrt{5}\,\overline{PH} = \sqrt{5}\cdot\frac{2}{\sqrt{3}} = \frac{2\sqrt{5}}{\sqrt{3}}$

$\therefore\ 2a = \overline{PA}+\overline{PB} = \frac{3\sqrt{5}}{\sqrt{3}} = \sqrt{15}$ ， $a = \frac{\sqrt{15}}{2}$

$\Rightarrow b^2 = \frac{15}{4}-\frac{3}{4} = 3$ ，中心$(0,0)$，左右型

$$\therefore\ \frac{x^2}{\frac{15}{4}}+\frac{y^2}{3} = 1$$ ※