

101 關西

一、填充

9.  $y = \tan x \sec x$ ,  $0 \leq x \leq \frac{\pi}{3}$  與  $x$  軸所圍的區域為  $\Gamma$   
 $\Gamma$  的面積為  $a$ ,  $\Gamma$  繞  $x$  軸的旋轉體體積為  $b$ ,  
則數對  $(a, b) = \underline{\hspace{2cm}}$ .

Sol:

$$a = \int_0^{\frac{\pi}{3}} \tan x \sec x \, dx$$

$$= \int_0^{\frac{\pi}{3}} \frac{\sin x}{\cos^2 x} \, dx$$

$$= - \int_0^{\frac{\pi}{3}} \frac{1}{\cos^2 x} \, d(\cos x) \quad \left( \because (\cos x)' = -\sin x \right)$$

$$= 1$$

$$b = \pi \int_0^{\frac{\pi}{3}} (\tan x \sec x)^2 \, dx = \pi \int_0^{\frac{\pi}{3}} \tan^2 x \sec^2 x \, dx$$

$$= \pi \int_0^{\frac{\pi}{3}} \tan^2 x \, d(\tan x) \quad \left( \because (\tan x)' = \sec^2 x \right)$$

$$= \pi \left[ \frac{1}{3} \tan^3 x \Big|_0^{\frac{\pi}{3}} \right]$$

$$= \sqrt{3} \pi$$

故  $(a, b) = (1, \sqrt{3} \pi)$   
✘